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6TH EDITION

Principles of Radiographic Imaging

AN ART AND A SCIENCE

Richard R. Carlton
Arlene M. Adler
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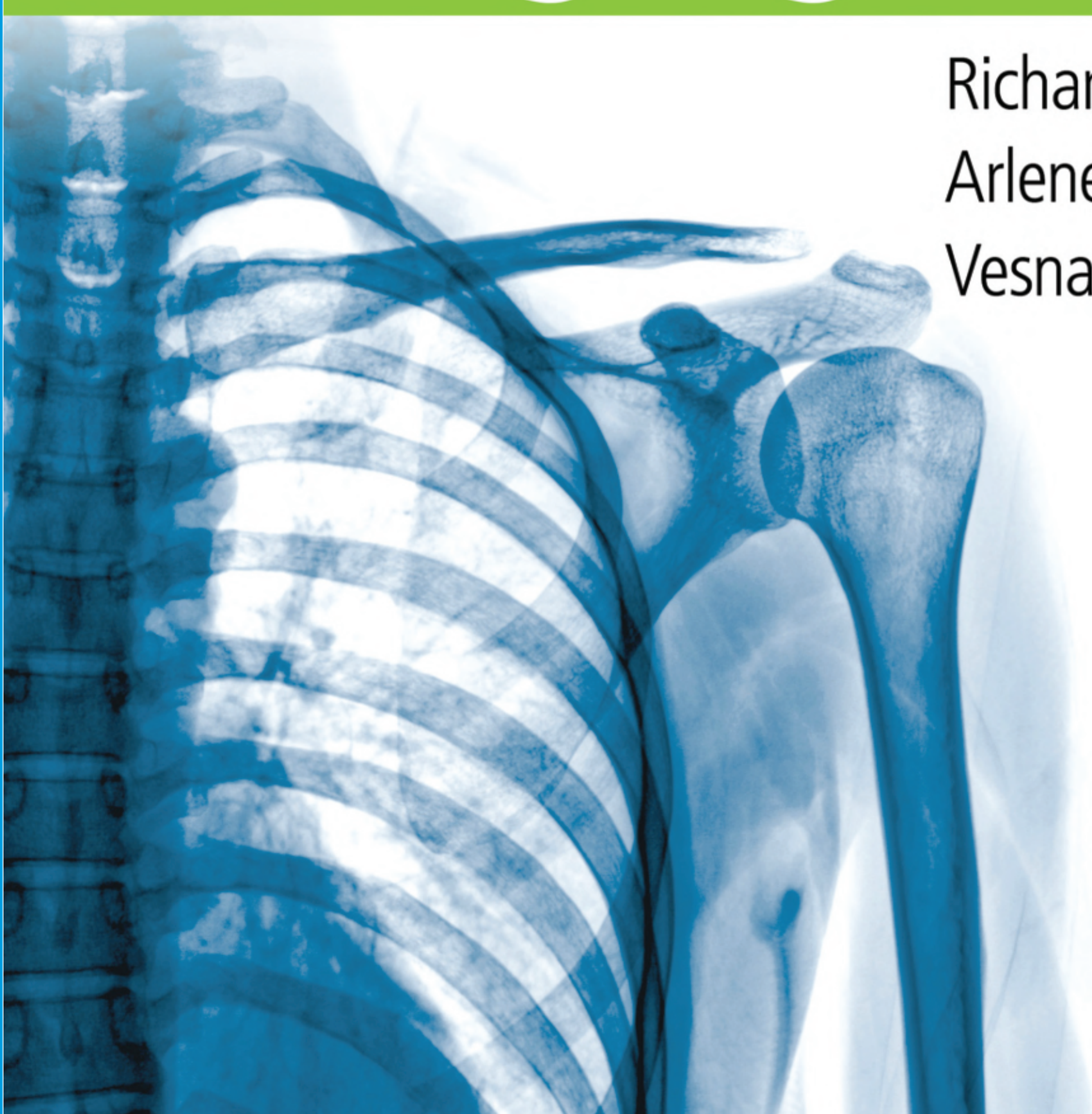

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Principles of Radiographic Imaging

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***Principles of Radiographic Imaging: An Art and a Science,* Sixth Edition**

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Dedication

In memory of Raymond P. Rossi.
Our mentor, friend, teacher,
and coauthor. We miss you, Ray.
To Sidney Finkleman and Quentin Garlets for
teaching us to be radiographers, to LaVerne
Ramaeker, Kay Shriver, and our students
for teaching us to be teachers,
and, most importantly,
to our families and friends
for sustaining us throughout.

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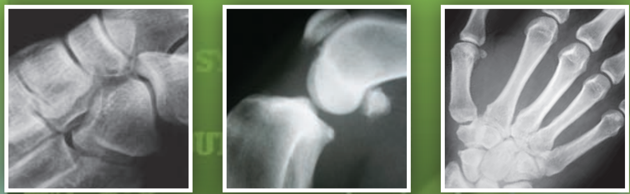


The Founders of Radiography

Wilhelm Conrad Röntgen (1845–1923)

Wilhelm Conrad Röntgen became the first radiographer when he discovered x-rays on November 8, 1895, in his laboratory at the University of Würzburg in Germany. His original paper *On a New Kind of Rays* is printed in Appendix A. For his discovery he received the first Nobel Prize in physics in 1901 and was decorated by Prussia, Bavaria, Great Britain, Austria, Mexico, Germany, France, the Netherlands, Sweden, Italy, Turkey, and the United States.

(Portrait courtesy of the American College of Radiology)

Eddy Clifford Jerman (1865–1936)

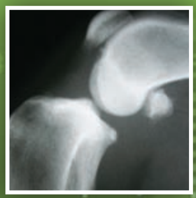
Ed C. Jerman is known as the "Father of Radiography" in the United States because he was the first teacher of radiography. He began teaching x-ray techniques in 1897; he founded the American Association of Radiological Technicians (now known as the American Society of Radiologic Technologists), and personally examined the first 1,000 members of the American Registry of X-Ray Technicians. He brought order to the principles of radiographic exposure technique by naming the qualities of the radiographic image: density, contrast, detail, and distortion. In 1928 he published *Modern X-Ray Technic*, the first book on radiographic principles.

(Portrait courtesy of the American Society of Radiologic Technologists)

Arthur Wolfram Fuchs (1895–1962)

Arthur W. Fuchs is known as the inventor of the fixed kilovoltage technique of radiography. His father, Wolfram Fuchs, established the first x-ray laboratory in Chicago in 1896 but became one of the early martyrs of radiation, dying from excessive exposure. Arthur performed radiography for the U.S. Army in both World Wars I and II. During World War II he wrote the U.S. Army training manual *Principles of Radiographic Exposure*, in which he outlined his success with the optimal fixed kVp technique system. In 1955 he first published his book *Principles of Radiographic Exposure and Processing.*

(Portrait courtesy of the family of Arthur Fuchs)

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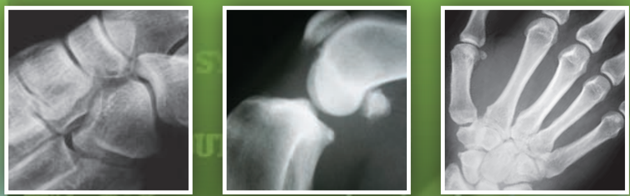

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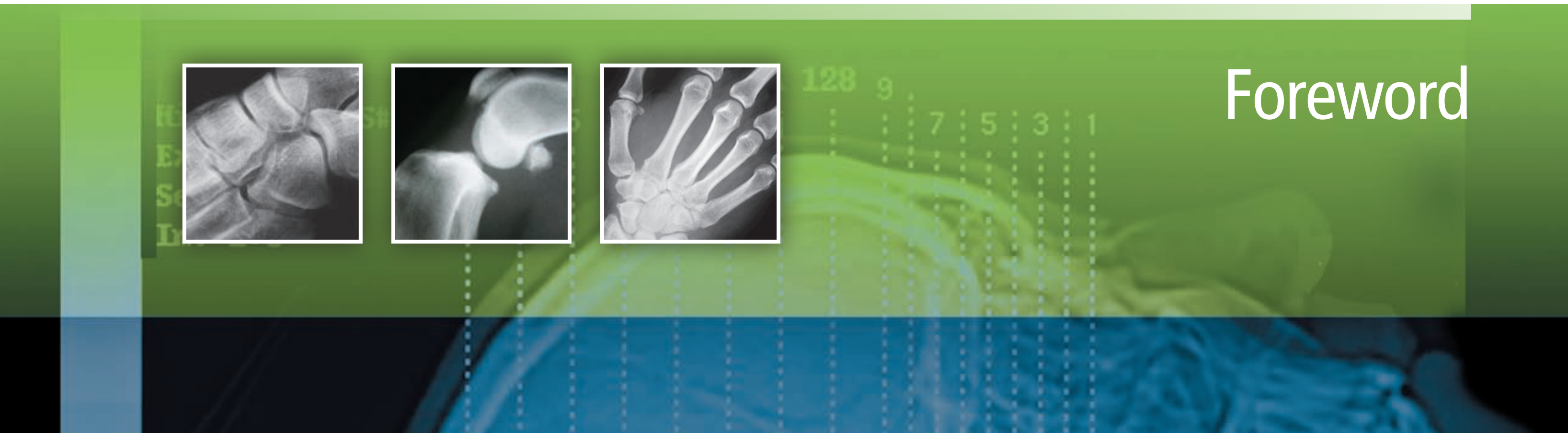

Foreword

Ohhhh, was my mother right!

Life is about making choices, and a good life is about making more right than wrong choices. Yes, a bit of good fortune and happenstance is in all of our lives, but ultimately, making good decisions is what it's all about. Unfortunately, at the times we make these choices, we lack the foresight to see the consequences, good or bad. But such is life.

Fortunately for our profession, Rick Carlton and Arlene Adler have continued to make the right decisions, by authoring *Principles of Radiographic Imaging*, 6th edition. It is a monumental task writing a textbook and to do so again, after so many years in the profession, makes it even more daunting. But that is the decision they have made and it is a great choice. Even better, is their decision to seek out experts and resources in the professions, such as Vesna Balac. Doing so complements the accuracy and relevancy of the information written in many of these chapters. That choice reflects their admission that one, single person cannot be expert on all aspects of this great profession. It is a tribute to their many years of developing a network of contacts who can contribute to this end. Again, their great decision-making over the years, has created a wide network of friends and resources, willing to contribute to this edition. Such is the nature of this 6th ed. It reflects their genuine and sincere effort to represent the principles of radiology in a real-world, practical fashion.

This 6th edition is faithfully consistent with previous editions through its excellent illustrations and exhibits. Updated materials for instructors with laboratory exercises and instructor resources are included as well. For the educator, you can expect to continue to enjoy its simple and concise explanations of complex concepts. New chapters on digital radiographic imaging have been added and existing chapters have been updated with practical information that is relevant to the real-world of digital radiography. In fact, much of this information could be studied by practicing radiographers, to update their working knowledge. With newer digital technologies, the old rules of exposure and image quality do not apply. Seasoned-technologists in the profession find this concept hard to accept. The 6th edition helps with this understanding. Topics dealing with the role of radiographic grids, detector interfacing, exposure indices and assessing digital image quality, have been added to better understand that as this digital revolution continues, we need to get our arms around it, to improve image quality and ALARA compliance. The role of the radiographer is undergoing profound change with digital, and a renaissance is underway through the applications of digital detectors.

This 6th edition makes the learning more interactive and exciting, through the use of web-based technology. Existing chapters on the advanced imaging modalities still remain with updated information from experts in their respective fields. A wonderful choice.

As my mother said, it's about choices. We are grateful Rick and Arlene, and now Vesna, have chosen to write again. In doing so, they introduce potentially new resources to educators, technologists and all members of the medical imaging community: the best profession in medicine!

To those who know me, they would say I am a passionate educator, as my passion for this professions runs deep. I am no different than many other professionals, in this regard. Incorporating the *Principles of Radiographic Imaging*, 6th edition into my professional lifestyle, is my choice, and for that I cannot thank these three authors enough for their continued contributions.

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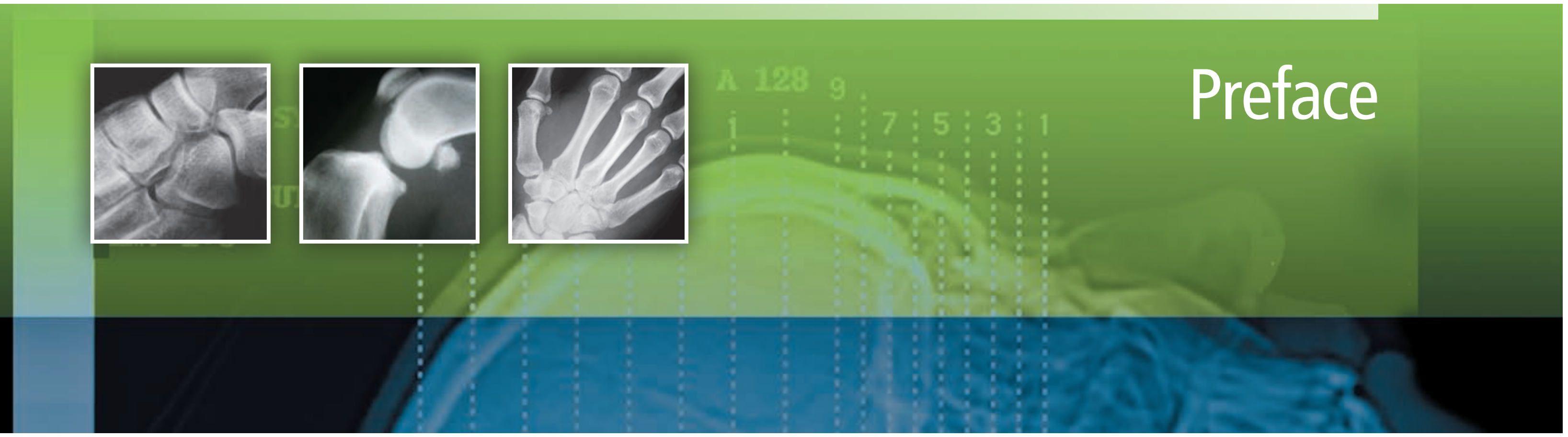


Preface

INTRODUCTION

As radiography educators, we designed this textbook with students and educators in mind. Since we now have a completely revised approach from a digital standpoint, we also believe practicing radiographers would greatly benefit from a review since there are new principles toward establishing radiographic exposure techniques. Each chapter contains an outline, key terms, objectives, a summary, review questions, and a detailed bibliography. There is also a considerable number of ancillaries that faculty can access through their Cengage Learning representative (they can be reached through the website at www.cengage.com, or for product information and technology assistance, contact them at Cengage Learning Customer and Sales Support, 800-354-9706.). Although the order of the chapters is based on our experiences in reaching our students, most chapters stand alone and can be used in the order that is most appropriate within a given program.

We have made a special effort to represent our belief that professional development should be a prime objective of any radiography curriculum. This is best achieved through the demonstration of technical competence. This book is designed to assist students in developing this cornerstone of professionalism. Through technical competence and a professional demeanor, students will be ready to assume their role as experts in the radiographic imaging process. In addition, we believe that true professionals take immense pride in their work to the extent that it becomes an art as well as a science.

There is a special focus on making sure digital imaging terminology and parameters are the primary orientation with very limited references to conventional film-based imaging. Special effort is made to emphasize both the technical information and the ethical importance of understanding the specifics of image receptor sensitivity in order to be able to reduce exposure to the patient and adhere to ALARA concepts.

References are used extensively to assure both educators and students that we address all content expected for successful professional practice including the American Registry of Radiographic Technologists' Content Specifications for the Examination in Radiography, the Radiography Curriculum by the American Society of Radiologic Technologists, and various materials prepared by the Canadian Association of Medical Radiation Technologists.

NEW TO THIS EDITION

The rapid changes in technology present a challenge to textbook authors who are committed to providing current information for learners. The authors and contributors to the sixth edition carefully reviewed all content to identify areas requiring updating or new topics. As a result, numerous changes were made.

A special attempt was made to provide an introduction to physics and the imaging modalities, as well as to explore the new details of the principles of radiographic exposure techniques that have been brought on by the use of digital technology. Well-established image quality factors (density/image receptor exposure, contrast, recorded detail, and distortion) have been revised to reflect the realities of establishing radiographic exposure factors. The new factors are:

- Image Receptor Exposure
- Contrast
- Spatial Resolution
- Distortion
- Histogram Fulfillment
- Look Up Table (LUT)

Because histogram fulfillment and use of the proper look up table (LUT) are critical to successful digital image production, they have both been included in the primary image quality factors.

Five phases of digitized image production are introduced: acquisition, processing, archiving, displaying, and analyzing. Each includes well-known radiographic imaging parameters but each is also enhanced by the addition of digital factors that now impact on all imaging decisions.

Most importantly, this entire edition has been revised with current clinical practices in mind, with many changes that now reflect our digital radiography world, from end to end.

ORGANIZATION

The overall design of the book separates the 42 chapters into six units: Creating the Beam, Protecting Patients and Personnel, Creating the Image, Digital Radiography, Analyzing the Image, and Special Imaging Systems and Modalities. Unit IV, Digital Radiography, has been completely rewritten into 5 redesigned chapters; Digital Image Processing, Computed Radiography, Digital Radiography/Flat-Panel Detector Systems, Technical Considerations in Digital Imaging, and Informatics in Medical Imaging. In addition, Unit V Analyzing The Image now reflects the new paradigm for technical exposure factors; Image Receptor Exposure, Contrast, Spatial Resolution, and Distortion. We continue to offer framework information on all radiologic and imaging sciences modalities and treatments. This design helps organize the content for students by following a logical progression from introductory physics through the production and control of the beam to advanced modality systems.

We remain extremely pleased that our book remains one of the resources listed in the Radiography Curriculum of the American Society of Radiologic Technologists, and we have long been pleased that this book continues to be recommended by the Canadian Association of Medical Radiation Technologists for preparation for its certification in Radiological Technology.

FEATURES

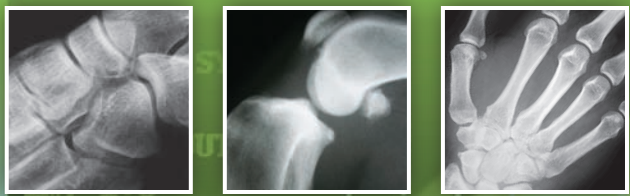
In addition to the updated and new content, this new edition continues to feature the following learning aids and critical content, with the addition of supporting video clips for the new digital radiography chapters:

- Physical concepts are clearly explained and illustrated with many high-quality full color figures.
- Effects of changing parameters on image quality are carefully described and illustrated with numerous images.
- Criteria for image analysis are presented to help learners develop analytical skills.
- High-quality radiographs are included throughout the text.
- Radiation protection concepts and procedures are emphasized for both patients and radiographic personnel.
- Chapter-end summaries provide a quick reference to critical concepts and developments in the science of radiography.
- Numerous troubleshooting tips are included to ensure quality radiographs.
- Extensive references and recommended readings provide a historical perspective and provide learners a means to expand their understanding of concepts and systems.
- Video clips are now available for the chapters on digital image production.
- Epigraphs and historical photos help trace the evolution of radiography to the present.
- Unique emphasis on the art versus the science of radiography illustrates the broad applications of the technology.

STATEMENT OF CONTENT ACCURACY

Although we assume full responsibility for any errors, including those that may be construed as arising from quoting other works out of context, we have made every effort to ensure the accuracy of the information. However, appropriate information sources should be consulted, especially for new or unfamiliar procedures. It is the responsibility of every practitioner to evaluate the appropriateness of a particular procedure in the context of actual clinical situations. Therefore, neither the authors nor the publisher take responsibility or accept any liability for the actions of persons applying the information contained herein in an unprofessional manner. This information is designed to supplement and enhance the instructional methodologies of educators in JRCERT (Joint Review Committee on Education in Radiologic Technologies [USA]) and CAMRT (Canada) approved radiography programs and should not be applied, especially to human subjects, without this background. In committing this book to print, we fully realize that it is never finished, merely suspended for the time being.

Finally, as a reader your perceptions are important to us. We encourage you to communicate with us regarding facets of the book you appreciate or would like to see changed. We especially appreciate constructive comments and notice of errors. Our intention is to present the principles of radiography in an interesting format that provides a base from which true professionalism can develop. Any commentary readers care to make toward this end will be valued and welcomed.



About the Authors

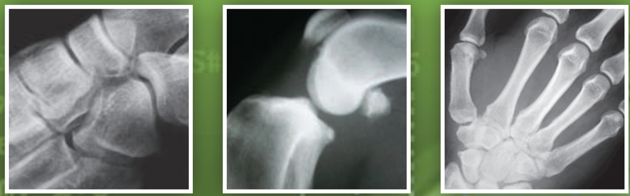
Richard R. Carlton is a Former Chair and Associate Professor of Radiologic and Imaging Sciences at Grand Valley State University in Grand Rapids, Michigan. He is a past President and Chairman of the Board of the Association of Educators in Imaging and Radiologic Sciences. Rick co-founded the Michigan Radiologic and Imaging Sciences Consortium (a consortium of community colleges and a university that offers advanced clinical education), and established the Center for Medical Imaging in Bioanthropology (for field work x-raying mummies in Peru), Lambda Nu (the national honor society for the radiologic and imaging sciences), and The Sectional Anatomy Consortium. He has published more than 20 books, was the founding editor of two journals, and is a charter Fellow of the Association of Educators in Radiologic Sciences (AEIRS). Rick has been a JRCERT accreditation site visitor for more than 30 years, and was a Fulbright Scholarship alternate, as well as author of numerous journal articles. He has given more than 250 lectures in 36 states and 9 countries.

He holds an M.S. from National Louis University, B.S. in Radiologic Sciences from The Chicago Medical School, and an A.A.S. from Illinois Central College. He is ARRT certified in radiography and cardiovascular-interventional technology and has taught radiologic sciences for over 30 years in Chicago, Ohio, San Francisco, Arkansas, and Michigan. With Arlene Adler, he is the author of the best-selling radiographic physics textbook *Principles of Radiographic Imaging* as well as *Introduction to Radiography and Patient Care*, both now in fifth editions. Rick has also been the radiology section consultant to *Taber's Cyclopedia Medical Dictionary* for the past three editions.

Arlene Adler is Professor Emerita, Radiologic Sciences Programs at Indiana University Northwest in Gary. She received her radiography education at Michael Reese Medical Center in Chicago. She has a B.S. degree in Radiologic Sciences from the University of Health Sciences/Chicago Medical School and a Master's Degree in Education from the University of Illinois at Urbana–Champaign. Arlene has been an active member of the American Society of Radiologic Technologist and the Association of Educators in Radiologic Sciences (AEIRS). For her service to AEIRS, she was awarded Fellow status. She has lectured across the country on numerous topics in radiologic sciences and, in particular, was a past Honors Lecturer for an AEIRS Annual Meeting and a past Keynote Speaker for the Associated Science Program at the Radiologic Society of North America (RSNA) annual meeting. For her service on the Indiana Radiation Control Advisory Commission, she received the Governor's Award, Sagamore of the Wabash.

In addition to her professional service, Arlene has authored numerous articles and, along with Rick Carlton, is coauthor of the text *Principles of Radiographic Imaging*: *An Art and a Science* and coeditor of the text *Introduction to Radiography and Patient Care*.

Vesna Balac is a Clinical Assistant Professor and Director of the Radiologic Sciences Programs at Indiana University Northwest in Gary, Indiana. She received her radiography education as well as her B.S. degree in Radiologic Sciences at Indiana University Northwest. She has a Master's Degree in Adult Education from Indiana University in Bloomington and is currently a doctoral candidate in Instructional Systems Technology at the School of Education at Indiana University Bloomington. She is ARRT certified in radiography and magnetic resonance imaging and an active member of the American Society of Radiologic Technologist (ARRT), the Association of Educators in Radiologic Sciences (AEIRS), and the Indiana Society of Radiologic Technologists (ISRT). In addition to her professional service, Vesna is a contributing author to the text *Introduction to Radiography and Patient Care*.



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The sixth edition has been produced by our most dedicated team at Cengage Learning.

We are especially grateful to Joe Chovan, the most spectacular artist we have ever worked with. Joe's artistic skills are on exhibit throughout the many editions of this book, as he allows us to bring new insight into complex content through his sparkling and insightful illustrations.

We are indebted to the unstinting contributions from Randy Griswold from Bellin College in Green Bay, Wisconsin. A significant portion of the accuracy of the book derives from the constant inspection and contributions he has made. Randy Griswold has been an enthusiastic and regular contributor to this text for many years and his additions and updates have been greatly appreciated. Dennis Bowman from the Community Hospital of the Monterey Peninsula in Monterey, California worked with us in his facility to refine digital techniques as well as prepare our video clips and illustrations. We have also been very appreciative of the constant input of Denise Moore, Professor Emeritus at Sinclair Community College in Dayton, Ohio, who has been our constant conscience from the first edition. We also cannot forget the contributions made to the first through fourth editions by Barry Burns of the University of North Carolina, who routinely went far beyond our wildest hopes in critiquing our text, producing films to illustrate his points, disproving old wives tales in his laboratory, and generously sharing his results (and venison) with us. Barry became a backbone of the technical aspects of this book. Eugene Frank, formerly of the Mayo Clinic and Riverland Community College in Minnesota, continued his persistent and knowledgeable critique of our efforts. We are grateful for Gene's unique contributions, especially his ability to never let nearly two decades of friendship come between us and a more accurate reworking of verbiage or the details of an illustration.

We acknowledge detailed contributions from John Skinner at Mid Michigan Community College; Jeff Lloyd at Spectrum Butterworth in Grand Rapids, Michigan; John Godisak at Grand Rapids Community College, Michigan; and Jennifer Lockhart from Saint Anthony Medical Center in Crown Point, Indiana.

The students, faculty, and staff at Arkansas State University played a major role in the third edition. Special thanks were due Dean Susan Hanrahan for unremitting support and constant enthusiasm for what we are trying to accomplish in the radiologic sciences.

We thank the faculty and students who have found our work valuable in the process of radiography education and practice since the first edition was published. The success of this book and the invitation to produce a new edition are a direct result of the acceptance of our work by the radiography profession, which we gratefully acknowledge.

Our families once again deserve our thanks for their understanding. Don Adler, Lynn Carlton, Louis Stevovich, and Mirko, Zorka, and Milorad Balac have given countless hours that were rightfully theirs, for which we extend a peace offering of love. Much appreciation is also extended to Arlene's daughters Meri and Katie, and Vesna's daughter Emili for their love and support. Rick's family now includes not only his children but his growing group of grandchildren: Nate, Michael, Zach, Mercie-Marie, Max, the twins Paige and Addison, Silas, Henry, and Penelope, all of whom get their time, book or not.

Once again, we have been assisted in our work by a wide spectrum of colleagues in the radiologic sciences and related fields. In addition to those who assisted us during the earlier edition, we wish to add our colleague Dennis Bowman from the Community Hospital of the Monterey Penisula; Euclid Seeram of British Columbia Institute

of Technology, Irven Rule of the Siemens Training and Development Center in Cary, North Carolina; Alfred Hufnagl of the Northern Alberta Institute of Technology, Canada; Penelope Roberts of the Department of Medical Physics and Medical Engineering at Southampton University Hospitals, UK; Kathheryn Root of Holyoke Community College, Massachusetts; Dr. Appel and Kevin Sisak at DuPont; Garry Harris at Agfa; and Gregory Wheeler of Wheeler and Associates, San Francisco, California.

Our faculties have also made significant contributions through their consistent willingness to comment on the countless details we have explored with them over the years. Alonso Contreras Astorga from the Department of Chemistry at Indiana University Northwest, also needs our thanks for his review of our basic explanations of chemistry concepts.

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And, finally, we thank those fellow educators who have taken the time to sit down with us at meetings, write letters, make phone calls, compose and email their comments and suggestions to us. Although we are certain we have not remembered all of you (for which we apologize), these include Mike Madden of Hays State University; John Clouse of Owensboro Community College, Kentucky; Marianne Tortorici and Mike Mixdorf at the University of Nevada, Las Vegas; Bob Misiak, Orange County Community College, New York; Jack Thomas at Lakeland Community College, Ohio; Lisa Iacovelli at Crozer-Chester Medical Center, Pennsylvania; Max Grady at Kettering College of Medical Arts, Ohio; Donna Mitchell at John Peter Smith Hospital, Texas; Anita Slechta of California State University at Northridge; Elwin Tilson of Armstrong State College, Georgia; Shay Mercer at New Mexico State University; Judy Williams of Grady Memorial Hospital, Georgia; Marilyn Sinderbrand of Northern Virginia Community College; Bill Sykes of Shawnee State University, Ohio; Bart Schraa of Daniel Denhoed University, Rotterdam, Netherlands; Frank Porter at St. Vincent Infirmary Medical Center Arkansas; Mitchell Bieber, University of Virginia Medical Center; Donna Foster at Northern New Mexico Community College; Bill May of Iatwamba Community College, Mississippi; Lorraine Henry of Orange Coast Community College, California; and Steve Dowd of the University of Alabama at Birmingham.

Appreciation is also expressed to the colleagues who reviewed the manuscripts for this edition. Their critical reviews helped to guide us in the preparation of the final manuscript.

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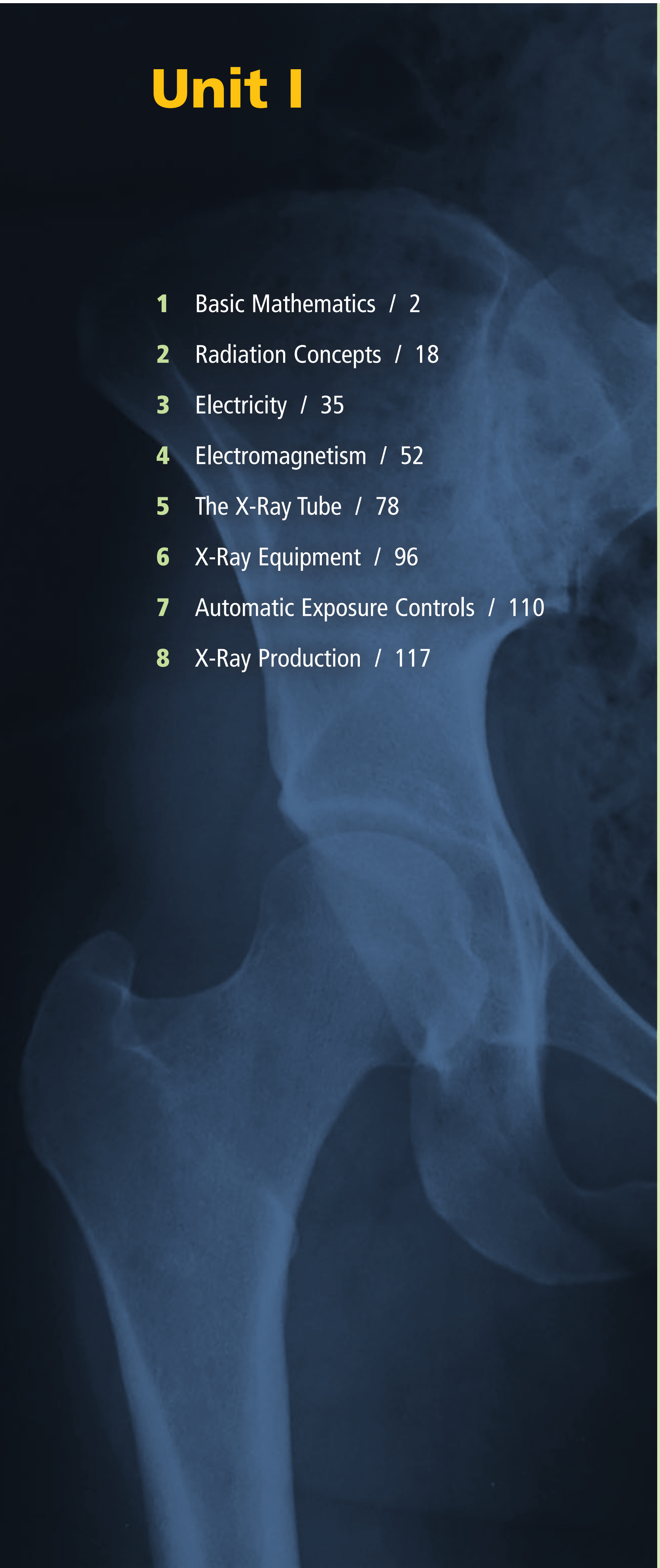
Unit I

Creating the Beam



An inherent quality of a professional is the possession of expertise regarding the technical aspects of a field far above that of a nonprofessional. Knowledge of the principles of radiographic image production is part of the technical expertise of the professional radiographer. Radiography programs provide students with classroom instruction, laboratory experience, and clinical practice in this subject. No other medical professional experiences as intensive or comprehensive a study of radiographic imaging. This unit is designed to provide the basics necessary for this knowledge by building a framework of information regarding the creation of the diagnostic x-ray beam.

The framework begins with an elementary review of **basic mathematics** and **radiation concepts**, including atomic theory, x-ray properties, and necessary units of measurement. Although this may be a review of previous science coursework for many readers, it is important to make sure everyone is on the same wavelength before using the information in the remainder of the book. **Electricity** and **electromagnetism** are large chapters that lay the foundation for understanding how to control the beam. The **x-ray tube**, **x-ray equipment, automatic exposure controls**, and **x-ray production** provide an understanding of exactly how basic physics is used to create the x-ray beam.

CHAPTER 1

Basic Mathematics

KEY TERMS

absolute value
algebraic expression
coulomb per kilogram (C/kg)
direct variation
equation
equivalent equation
gray (Gy)
identity
inverse variation
kilogram
like term
meter
open equation
second
sievert (Sv)

> Do not worry about your difficulties in mathematics. I can assure you mine are still greater.
>
> *Attributed to Albert Einstein*

OBJECTIVES

Upon completion of this chapter, the student should be able to:

- Perform functions with fractions and decimals.
- Determine significant digits in a number.
- Perform calculations in scientific notation with signed numbers and exponents.
- Simplify algebraic expressions.
- Convert units within the SI system.

THE LANGUAGE OF SCIENCE

Mathematics is the language of science. Radiographers need to be able to speak this language. In order to quantify science, standard units of measurement were established. The fundamental units of measurement are mass, length, and time. Units of measurement were officially defined on an international level through the adoption of the SI unit system. The seven base SI units are mass, length, time, electric current, temperature, amount of substance, and luminous intensity. Radiologically important derived units are the coulomb per kilogram (C/kg), formerly the roentgen (R); the gray (Gy), formerly the rad (radiation absorbed dose); and the sievert (Sv), formerly the rem (radiation equivalent man). This review is intended to refresh essential skills in the use of math as well as appropriate units for the radiologic and imaging sciences.

ARITHMETIC

The radiologic technologist must have a basic understanding of computations such as addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals.

Fractions

A fraction is a portion of a whole number and represents parts of a whole. The top number in a fraction is the numerator and the bottom number is the denominator.

Addition and Subtraction. To add or subtract two fractions with **like** denominators, add or subtract the numerators and keep the like denominator.

EXAMPLES:

$$\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$$

$$\frac{7}{8}-\frac{4}{8}=\frac{3}{8}$$

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$$

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

To add or subtract two fractions with **unlike** denominators, rewrite each fraction with a like or common denominator. Then add or subtract the numerators and keep the like or common denominator.

EXAMPLES:

$$\frac{1}{4}+\frac{5}{6}=\frac{3}{12}+\frac{10}{12}=\frac{13}{12}$$

$$\frac{7}{5}-\frac{1}{3}=\frac{21}{15}-\frac{5}{15}=\frac{16}{15}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

$$\frac{a}{d}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$$

Multiplication. Multiplication can be written in several ways (Table 1-1). To multiply two fractions, multiply the numerators and multiply the denominators.

EXAMPLE:

$$\frac{2}{5}\times\frac{3}{7}=\frac{2\times 3}{5\times 7}=\frac{6}{35}$$

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

Division. To divide two fractions, rewrite the division problem as a multiplication problem by multiplying the first fraction by the second fraction inverted.

EXAMPLE:

$$\frac{2}{5}\div\frac{3}{7}=\frac{2}{5}\times\frac{7}{3}=\frac{2\times 7}{5\times 3}=\frac{14}{15}$$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{a\times d}{b\times c}$$

TABLE 1-1. Multiplication Notation

Throughout this review the algebraic notation for multiplication will be used.
Example: 3×4 will be written as $3 \cdot 4$.
A · (dot) is used as a symbol for multiplication.
Example: $a \times b$ will be written as $a \cdot b$ or as ab.
When two letters are used, the dot is usually omitted.
Example: $3 \times a$ will be written as 3a or as $3 \cdot a$.
Parentheses can also be used to represent multiplication.
Example: 3×4 can be written as (3)(4).

Decimals

Decimal number place value (columns) is determined as shown in Figure 1-1.

Addition and Subtraction. To add or subtract two decimal numbers, line up the decimal points in each number, adding or subtracting as with whole numbers. Remember to add in zeros to fill out the decimal positions, if necessary. The decimal point must remain in the same position.

EXAMPLES:

Add:	***Subtract:***	***Rewrite as:***
76.81	76.1	76.10
+ 384.1	− 2.96	−2.96
460.91		73.14

Multiplication. To multiply two decimal numbers, multiply the numbers as if they were whole numbers. Place the decimal point in the product so the number of places in the product equals the sum of the number of decimal places in each number.

EXAMPLE:

Multiply:

2.31	2 decimal positions
× 6.8	1 decimal position
1848	
1386	
15.708	3 decimal positions

Division. To divide two decimal numbers, set up as if doing whole number division. Move the decimal point in the divisor to the right to make the divisor a whole number. Move the decimal point in the dividend the same number of places to the right (as was done for the divisor). Add zeros if necessary to maintain the position of the decimal. Divide the numbers as if they were whole numbers. Place the decimal point in the quotient directly above the decimal point in the dividend. Add zeros if necessary to maintain the position of the decimal.

EXAMPLES:

$0.25\overline{)1250}$

$$\begin{array}{r} 5000. \\ 25.\overline{)125000.} \end{array}$$

$1.3\overline{)733.2}$

$$\begin{array}{r} 564. \\ 13.\overline{)7332.} \\ \underline{65} \\ 83 \\ \underline{78} \\ 52 \\ \underline{52} \\ 0 \end{array}$$

Convert a Fraction into a Decimal Number. To convert a fraction into a decimal, divide the denominator into the numerator.

EXAMPLE: Convert 7/8 into a decimal:

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \end{array}$$

Decimal number place value (columns)

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths
1,000,000	100,000	10,000	1,000	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$	$\frac{1}{10,000}$

FIGURE 1-1. Decimal number place values.

Convert a Percent to a Decimal. To convert a percent to a decimal, move the decimal point two places to the left.

EXAMPLE: Convert 78.5% to a decimal:

78.5% = 0.785

Convert a Decimal to a Percent. To convert a decimal to a percent, move the decimal point two places to the right.

EXAMPLE: Change 0.452 to a percent:

.452 = 45.2%

Computation with Values (Numbers)

Exact. Some numbers are exact values, like the counting numbers (1, 2, 3, . . .). For example, there are exactly four legs on a chair. Exact numbers in computations are as accurate or as precise as needed.

Significant Digits. Some values are obtained by measurement and are only as accurate as the measuring device. The last digit in the reading is usually estimated. **When dealing with these values in computations, the results will only be as accurate as the least accurate value.** This is the concept of significant digits.

To determine the number of significant digits in a value (number):

1. Count all nonzero digits.
2. Count all zeros between nonzero digits.
3. Count all zeros at the end of a decimal value.

The number of significant digits in a value is the sum of the numbers obtained in steps 1, 2, and 3.

EXAMPLES: The following are measured values from a radiation dose meter (dosimeter):

7.14 mR has three significant digits.

90.104 mR has five significant digits.

0.048 mR has two significant digits.

7300 mR has two significant digits.

6.900 mR has four significant digits.

Precision. The precision of a value refers to the decimal position of the last significant digit.

EXAMPLE:

0.0218 is precise to the ten-thousandths.

Rounding Off. To round a number, the last digit to be retained is:

1. Left unchanged if the digit to the right of the last digit to be retained is less than 5.
2. Increased by one if the digit to the right of the last digit to be retained is 5 or greater.

EXAMPLES:

75.2581 rounded to 4 significant digits is 75.26.

6,836.66 rounded to 2 significant digits is 6,800.

0.0381 rounded to 2 significant digits is 0.038.

0.0299 rounded to 2 significant digits is 0.030.

Multiplication and Division of Approximate Values. When multiplying or dividing two or more approximate values, the number of significant digits in the final answer is no greater than the number of significant digits in the value with the least number of significant digits.

EXAMPLE: Using the calculator, (103.81)·(1.34) = 139.1054. However, because 1.34 has only three significant digits, the answer should have only three significant digits. Therefore, 139.1054 needs to be rounded to three significant digits, making it 139.

Addition and Subtraction of Approximate Values. When adding or subtracting two or more approximate values, the final answer should be no more precise than the **least** precise of the values.

EXAMPLE: Add the following:

$$\begin{array}{r} 123.1 \\ 89.123 \\ +103.3456 \\ \hline 315.5686 \end{array}$$

Because the least precise number is 123.1, the answer should be rounded to 315.6.

Powers of 10

The decimal system is based on powers of 10, as is the metric system. These are very important systems to the radiographer. When a^n is written, a is called the base and n is the exponent. Integers are defined as the numbers . . . −3, −2, −1, 0, 1, 2, 3, ...

Positive Integer Exponents. Definition:

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{\text{n times}} \text{ (multiply n times)}$$

EXAMPLES:

$10^1 = 10$

$10^2 = 10 \cdot 10 = 100$

$10^3 = 10 \cdot 10 \cdot 10 = 1000$

$10^n = 10 \cdot 10 \cdot 10 \cdots 10 = 100 \ldots 0$ (n zeros)

Negative Integer Exponents. Definition:

$$a^{-n} = \frac{1}{a^n} \text{(for a} \neq 0)$$

$$10^{-1} = \frac{1}{10} = .1$$

$$10^{-2} = \frac{1}{10^2} = .01$$

$$10^{-3} = \frac{1}{10^3} = .001$$

$$10^{-n} = \frac{1}{10^n} = .00\ldots001 \text{ (n} - 1) \text{ zeros}$$

Zero Exponents. Definition: $a^0 = 1$ (for $a \neq 0$)

EXAMPLE:

$10^0 = 1$

Scientific Notation

A number written in scientific notation is written as the product of a number between 1 (including 1) and 10 times a power of 10. In other words, $N \times 10^p$, where $1 \leq N < 10$ and p is an integer.

EXAMPLES:

7.15×10^4 is a number in scientific notation.

1.598×10^{-5} is a number in scientific notation.

7.58×10^0 is a number in scientific notation.

Converting Numbers from Ordinary to Scientific Notation. To write a number in scientific notation, the decimal point must be moved to the position following the first nonzero digit.

EXAMPLES:

$70.57 = 7.057 \times 10^1$ ($p = 1$)

$0.00815 = 8.15 \times 10^{-3}$ ($p = -3$)

$7.58 = 7.58 \times 10^0$ ($p = 0$)

In the first example, the decimal point is moved one digit to the left, making the power of 10 (p), 1. In the second example, the decimal point is moved three digits to the right, making the power of 10, −3. In the third example, the decimal point is not moved and the power of 10 is 0.

In general, when moving the decimal point to the left, the power of 10 will be the number of positions moved. When moving the decimal point to the right, the power of 10 will be negative the number of positions moved. When the decimal point is not moved, the power of 10 will be zero.

Converting Numbers from Scientific to Ordinary Notation. To convert a number in scientific notation to ordinary notation, the decimal position is moved according to the following:

1. If the power of 10 is positive, move the decimal point to the right the number of positions in the exponent.
2. If the power of 10 is negative, move the decimal point to the left the number of positions in the exponent.
3. If the power of 10 is zero, the decimal remains in the same position.

In all of the above, once the decimal point has been moved, the 10 and its power are dropped. Zeros are added, if necessary.

EXAMPLES:

$3.7 \times 10^4 = 37{,}000$

$5.56 \times 10^{-5} = 0.0000556$

$1.34 \times 10^0 = 1.34$

Dimensional Analysis

The concept of dimensional analysis is useful when converting from one set of units to another. The principle is based on fractions whose quotients are 1.

EXAMPLE:

$$\frac{12 \text{ in}}{1 \text{ ft}} = 1 \text{ or } \frac{1 \text{ ft}}{12 \text{ in}} = 1$$

Recall that multiplying a quantity by 1 does not change the value of the quantity. In order to convert from one set of units to another, algebraic operations are performed with units in the same way they are with algebraic symbols.

EXAMPLE: Convert 7.0 ft to inches.

$$7.0 \text{ ft} = 7.0 \cancel{\text{ft}} \cdot \frac{12 \text{ in}}{1 \cancel{\text{ft}}} = 7 \cdot 12 \text{ in} = 84 \text{ in}$$

Note that it is possible to "cancel" the feet unit, leaving the inches unit.

Convert 70.0 km/hr to m/s.

$$70.0 \frac{\text{km}}{\text{hr}} =$$

$$70.0 \frac{\cancel{\text{km}}}{\cancel{\text{hr}}} \cdot \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \cdot \frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ s}} =$$

$$70.0 \cdot 1000 \cdot \frac{1}{60} \cdot \frac{1 \text{ m}}{60 \text{ s}} = 19.4 \frac{\text{m}}{\text{s}}$$

Recall that 1 km = 1,000 m, 1 hr = 60 min, and 1 min = 60 s. Note that it is possible to "cancel" the km unit, the hr unit, and the min unit.

When converting cubic units or square units, extra attention needs to be given to the conversion factors. Study the following example carefully.

EXAMPLE: Convert 3.8000 ft^3 (cubic feet) to cubic inches (in^3).

$$3.8000 \text{ ft}^3 = 3.8000 \text{ ft}^3 \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{12 \text{ in}}{1 \text{ ft}}$$

$$= 6566.4 \text{ in}^3$$

or

$$3.8000 \text{ ft}^3 = 3.8000 \text{ ft}^3 \cdot \left(\frac{12 \text{ in}^3}{1 \text{ ft}}\right) = 6566.4 \text{ in}^3$$

EXAMPLE: Convert 10 roentgens per hour to roentgens per minute.

$$10.0 \frac{\text{roentgens}}{\text{hour}} = 10.0 \frac{\text{roentgens}}{\text{hour}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$= 0.167 \frac{\text{roentgens}}{\text{minute}}$$

ALGEBRA

Algebra uses symbols and specific rules of number manipulation to solve equations.

Signed Numbers

Numbers are not always positive. For example, the thermometer indicates that it is very cold at −20°F. Negative numbers can be illustrated by using a number line (Figure 1-2). Positive numbers are assigned to the right of the zero point and negative numbers to the left of the zero point. A statement that 7 is less than 10 (written as $7 < 10$) indicates that 7 is to the left of 10 on the number line. A statement that −5 is greater than −9 (written as $-5 > -9$) indicates that −5 is to the right of −9 on the number line.

Absolute Value. The **absolute value** of a number is the distance from 0 to that number on the number line. Because distance is the same number of units whether moving up or down the number line, that means that the absolute value of a number is always positive or zero. The notation used for absolute value is two vertical bars (| |).

EXAMPLES:

$|-2| = 2 \qquad |4| = 4 \qquad |0| = 0$

Because −2 is two units from zero on the number line, the absolute value is 2. Because 4 is four units from zero on the number line, the absolute value is 4. Because 0 is zero units from zero on the number line, the absolute value is 0.

Addition. To add two positive numbers, add their absolute values and attach a positive sign to the result. The positive sign is usually omitted.

EXAMPLE:

$(+7) + (+8) = +15 \qquad 7 + 8 = 15$

To add two negative numbers, add their absolute values and attach a negative sign to the result.

EXAMPLE:

$(-7) + (-8) = -15 \qquad -7 + (-8) = -15$

FIGURE 1-2. Number line.

To add two numbers with unlike signs, subtract the smaller in absolute value from the larger in absolute value and attach the sign of the larger in absolute value to the result.

EXAMPLES:

$(-10) + (3) = -7$ $\quad 4 + (-9) = -5$

$(8) + (-5) = 3$ $\quad -3 + (7) = 4$

Negative of a Number. The negative of a number is the opposite of the number in sign.

EXAMPLE:

The negative of 7 is -7; the negative of -7 is 7.

If $b = 7$, then $-b = -7$.
If $b = -7$, then $-b = -(-7) = 7$.

Subtraction. Subtraction is rewritten as addition, as in $a - b = a + (-b)$.

EXAMPLES:

$8 - 10 = 8 + (-10) = -2$
$8 - (-10) = 8 + 10 = 18$
$-8 - 10 = -8 + (-10) = -18$
$-8 - (-10) = -8 + 10 = 2$

All of the above subtraction problems are rewritten as addition and then the rules for addition of signed numbers are used.

Multiplication. To multiply two numbers, multiply their absolute values. If both numbers are positive or both numbers are negative, attach a positive sign to the result. If the numbers are opposite in sign, attach a negative sign to the result.

EXAMPLES:

$(2)(3) = 6$

$(-2)(-3) = 6$

$(2)(-3) = -6$

$(-2)(3) = -6$

Division. To divide two numbers, divide their absolute values. If both numbers are positive or both numbers are negative, attach a positive sign to the result. If the numbers are opposite in sign, attach a negative sign to the result.

EXAMPLES:

$8 \div 2 = 4$

$-8 \div -2 = 4$

$8 \div (-2) = -4$

$-8 \div 2 = -4$

Order of Operation

Order-of-operation problems may occur when parentheses are not indicated. For example, evaluate $2 + 3 \cdot 4$. Depending on whether addition or multiplication is performed first, the answer might be 20 or 14. The answer actually depends on the order of operation. Rules have been established that make the correct result 14.

The difference between the unary minus sign $(-)$ and the subtraction sign $(-)$ and the difference between the unary plus sign $(+)$ and the addition sign $(+)$ must be understood before order-of-operation rules can be learned.

A subtraction sign $(-)$ is used between two numbers.

EXAMPLE:

$7 - 8$ (subtraction sign)

A unary minus sign $(-)$ is used before one number.

EXAMPLE:

-7 (unary minus sign)

An expression can have both a subtraction sign and a unary minus sign.

EXAMPLE:

$-7 - 8$ (The first is a unary minus, the second is a subtraction sign.)

An addition sign $(+)$ is used between two numbers.

EXAMPLE:

$8 + 10$ (addition sign)

A unary plus sign (+) is used before one number.

EXAMPLE:

$+8$

Rules for Order of Operation

1. Perform all operations inside grouping symbols (parentheses, radical symbols, fraction bar, brackets, etc.).
2. Exponentiation (raising to a power or roots)
3. Unary minus (−) or unary plus (+)
4. Multiplication and division
5. Addition and subtraction

Operations are performed from the lowest level (#1) to the largest level (#5). Operations on the same level are evaluated left to right.

EXAMPLES:

$2 + \boxed{3 \cdot 4}$	(multiplication first)
$= 2 + 12$	(addition)
$= 14$	
$\boxed{8 \cdot 6} - \boxed{12 \div 2}$	(multiplication and division first)
$= \boxed{48 - 6}$	(subtraction)
$= 42$	
$\frac{8+7}{2+3}$	(bar acts as grouping symbol)
$= \frac{15}{5}$	(add numerator and add denominator first)
$= 3$	(divide)
$(-2)^3$	(exponentiation first)
$= -8$	
$-(2)^4$	(exponentiation first)
$= -(16)$	(unary minus sign)
$= -16$	
$3(\boxed{7 + 4})$	(add inside parentheses first)
$= \boxed{3 \cdot 11}$	(multiply)
$= 33$	
$2(\boxed{6 + 1})^2$	(add inside parentheses first)
$= 2\boxed{(7)^2}$	(exponentiation next)
$= 2 \cdot 49$	(multiply)
$= 98$	

Algebraic Expressions

An **algebraic expression** consists of letters and/or numbers that are multiplied, divided, added, subtracted, or raised to a power.

EXAMPLE:

$3x^3 + 5x^2 - 7x = 8$

$\frac{x+5}{y-7}$

There are several rules for combining algebraic expressions.

Distributive Law. The statement of the distributive law is:

$$a(b + c) = a \cdot b + a \cdot c$$

EXAMPLE:

$7(x + y) = 7x + 7y$

$7(3 + 5) = 7 \cdot 3 + 7 \cdot 5 = 21 + 35 = 56$

$p(x + 4) = px + 4p$ (The numeral is normally written in front of the letter.)

If the addition sign were a subtraction sign, then:

$$a(b - c) = a \cdot b - a \cdot c$$

EXAMPLE:

$7(x - y) = 7x - 7y$

$7(3 - 5) = 7 \cdot 3 - 7 \cdot 5$

$= 21 - 35 = 21 + (-35) = -14$

$p(x - 4) = px - 4p$

Care needs to be taken when one or two negative signs are involved.

EXAMPLE:

$-7(x + y) = -7x + (-7y)$	(distributive law)
$= -7x - 7y$	(the reverse definition of subtraction)
$-7(x - y) = -7x - (-7y)$	(distributive law)
$= -7x + 7y$	(definition of subtraction)
$-7(-3 - 8) = (-7)(-3) - (-7)(8)$	(distributive law)
$= 21 - (-56)$	(multiply)
$= 21 + 56$	(definition of subtraction)
$= 77$	

Addition and Subtraction of Like Terms. A **like term** is a term with identical literal factors. A literal factor is a factor denoted by a letter. Like terms may be added or subtracted.